PHYSICAL CONDITIONS OF HEAT TRANSFER AND THE DESIGN OF HEAT PIPES OPERATING IN THE EVAPORATION MODE AT MODERATE TEMPERATURES

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Analyzed are the physical conditions of heat transfer in heat pipes operating in the evaporation mode at moderate temperatures. An engineering design method is proposed for heat pipes.

Layout of Heat Exchanger Pipes and Their Principle of Operation. A heat pipe is considered which operates by the evaporation–condensation cycle. In a general layout of heat pipes for a continuous-process operation, the condensate returns to the evaporator either through gravity or pumping action. The pumping action in a heat pipe is provided by a wick of a special structure. A heat exchanger tube in this study will be represented as a hermetically enclosed cylindrical body, the inside wall lined with a porous material and the latter saturated with the working liquid. A heat pipe can be divided lengthwise into three segments: the evaporation zone (evaporator), the transport zone, and the condensation zone (condenser).

The working liquid evaporates in the evaporator when heat is supplied outside through the walls. The vapor flows through the wick into the inner pipe cavity and is transported to the condenser, where it condenses as a result of heat removal. The condensate returns to the evaporator by the action of capillary forces. The vapor is driven by the pressure difference between evaporator and condenser. The transport of liquid is effected by the pressure difference due to capillary forces. It follows from an analysis of the force balance that a heat pipe can operate only when all the pressure losses in the liquid and in the vapor together do not exceed the maximum possible capillary pressure, i.e., when

$$\Delta P_V + \Delta P_L \leqslant \frac{2\sigma_e}{R_e} . \tag{1}$$

Physical Conditions of Heat and Mass Transfer in a Heat Pipe. One must distinguish two modes of operation here, with different patterns of heat and mass transfer in the evaporator: the evaporation mode and the boiling mode [2]. In the first case vapor is generated only on the outside surface of the liquid–vapor interphase boundary. Heat through the wet wick is transmitted by conduction. In the boiling mode vapor is generated also inside the wick structure. The bubbles moving through the wick turbulize the liquid in it and increase the rate of heat transfer there. A survey of the technical literature reveals that the majority of researchers consider only the evaporation mode to be effective and pay most of their attention to it, with an almost complete disregard of the boiling mode [1, 3, 4]. At the same time, our own studies indicate that the heat transfer rate is higher during boiling and that this mode deserves much consideration.

A heat pipe is thermally "superconducting" on account of the molecular nature of the heat transfer. One may assume that the energy is transmitted through a heat pipe only as the latent heat of evaporation, proportional to the mass flow rate of liquid or vapor. This flow rate may be limited by several factors having to do with the peculiarities of heat and mass transfer processes within the various pipe segments. Generally, such limitations include the specific velocity of liquid feed to the evaporator, the maximum attainable local thermal fluxes in the evaporator and in the condenser, etc. In this article we will consider the basic limitation typical of heat pipes within the moderate temperature range, which has to do with the

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pressure losses in vapor and liquid due to viscous drag forces. These losses impede the supply of liquid to the evaporator and impose a so-called hydrodynamic limit on the heat transfer capability of heat pipes. Thus, the hydrodynamic limit determines the maximum evaporator load corresponding to the maximum allowable pressure losses in liquid or vapor. This limit will be determined here analytically for heat pipes operating in the evaporation mode.

Effect of Wick Structure on the Evaporation and the Transport of Liquid. A capillary-porous body consists of a solid matrix with pores of all possible sizes and shapes. For simplicity, we will consider a porous fiber wick with a criss-cross structure. A model of such a wick may be one consisting of several layers of metal meshes. Each layer can be characterized by the average fiber diameter $2r_i$ and the average eye width $2b_i$. A square mesh cell will be replaced by a circular one with the equivalent eye radius b_i. If the wick is initially saturated with liquid, then during evaporation as a result of added energy the liquid–vapor interphase boundary will shift deeper into the wick and will curve under the influence of surface tension. This boundary will continue penetrating deeper, until the difference between pressure in the condenser and pressure under the meniscus, i.e., the pressure difference which causes the boundary to move deeper becomes equal to the pressure losses due to viscous forces. In this state, then, the liquid—vapor interface can be characterized by the radius of curvature R, which is a function of the penetration depth or of the volume of evaporated liquid.

The relation between the radius of curvature R and the wick volume occupied by vapor is, for f layers of metal meshes making up a wick element of length $2(b_1 + r_1)$,

$$V = \sum_{i=0}^{i-1} \varepsilon_i F_i \cdot 2(b_1 + r_1) + m_i n_i \frac{1}{3} \pi R^3 \left\{ \left[1 - \cos\theta \sqrt{1 - \left(\frac{b_i + r_i}{r_i + R\cos\theta} \right)^2} \right]^2 \left[2 + \cos\theta \sqrt{1 - \left(\frac{b_i + r_i}{r_i + R\cos\theta} \right)^2} \right] \right\}$$
$$+ m_i n_i \frac{\pi r_i}{3} \left[1 - \sqrt{1 - \left(\frac{b_i + r_i}{r_i + R\cos\theta} \right)^2} \right]$$
$$\times \left[(b_i + r_i)^2 + R \left(\frac{b_i + r_i}{r_i + R\cos\theta} \right)^2 \cos\theta + \frac{R (b_i + r_i)^2 \cos\theta}{r_i + R\cos\theta} \right], \quad (2)$$

where m_i is the number of cells per length πd_V in the i-th layer, n_i is the number of cells in the i-th layer per one cell of the first layer (layer 1), and r_i is the wire radius in the i-th mesh.

The heat transmitting capacity of a tube is very much affected by the thermal conductivity of the wet wick. The thermal conductivity of a wet wick consisting of interwoven meshes of stainless steel wire is, to a definite extent, affected by the friction of the wick to the pipe walls, since the total thermal resistance varies widely with varying wick thickness as well as with varying contact between meshes and pipe walls. In our experiment the meshes were retained by a spring wound with d = 0.6 mm steel wire. The test data for meshes with a porosity $\varepsilon = 0.5$-0.75 have revealed that the thermal conductivity of such a wick approaches that of the working liquid.

In order to produce normal operating conditions in the evaporator, it is necessary that the mass flow of vapor from the evaporation surface and the mass flow of liquid from the condenser to the evaporator be in equilibrium at any heat load level. This condition can be attained by means of wicks with a cell size varying along the pipe radius.

The choice of a meshed wick depends on the operating mode in the pipe, by the operating temperature, by the kind of liquid, and by the pipe geometry, i.e., by factors which determine the limitations on heat and mass transfer. For heat pipes operating at moderate temperature levels, most effective are wicks whose element with the smallest pores is placed on the side of the heating surface. Such an arrangement yields not only smaller radial and axial temperature drops in the pipe, but also ensures a stable tube performance in the boiling mode without leaving the evaporator walls dry, even at thermal flux densities as high as 15-20 W per 1 cm^2 of heating surface.

Hydrodynamic Analysis of Heat and Mass Transfer in a Heat Pipe. The purpose behind the theoretical analysis of the processes occurring in a heat pipe is to determine the individual pipe parameters, namely: the vapor temperature and pressure, the wall temperature, the pipe and wick geometry, the maximum evaporator load, etc. Known methods of designing heat pipes, as proposed by several foreign authors, have some substantial drawbacks. Thus, the linearized Schindler–Woesner model [3] rests on a few rough assumptions concerning the thermal mode of operation in a pipe and the hydrodynamics of liquid flow through the wick. In the Cosgrove (and associates) model [4] the potential of liquid transport from condenser to evaporator is determined by gravitational forces. This assumption is not valid in many cases as, for example, in attenuated gravitation fields.

The model which we propose here is based on the following assumptions:

1) the heat pipe is cylindrical;

2) the thermal flux density is uniform along the pipe, but different in the evaporation zone and in the condensation zone;

3) the temperature of the pipe wall and the temperature of the liquid are uniform within each of the three zones: evaporation, transport, and condensation;

4) the flow of liquid through the wick is laminar and obeys Darcy's Law;

5) the flow of vapor and of liquid can be described by the Navier–Stokes equations for an incompressible fluid;

6) the problem is one-dimensional;

7) there are no gravitational forces present;

8) all the heat transmitted to the evaporator by conduction is spent on evaporation.

The continuity equation, the flow equation, and the energy equation for a pipe element of length dz are, respectively:

$$\varepsilon F_{\text{wick}} \frac{dG_{\text{L}}(z)}{dz} \pm \pi d_{\text{V}} G_{\text{V}} = 0, \tag{3}$$

$$\frac{1}{2} \rho_{\text{L}} \frac{d}{dz} [G_{\text{L}}^{2}(z)] = -\frac{dP}{dz}, \tag{4}$$

$$i_{\text{L}} \frac{dG_{\text{L}}(z)}{dz} \pm \frac{\pi d_{\text{V}}}{\varepsilon F_{\text{wick}}} i_{\text{V}} G_{\text{V}} = \pm \frac{Q}{\varepsilon F_{\text{wick}} L}, \tag{5}$$

where the plus sign and the minus sign refer to the evaporator and the condenser, respectively. The pressure gradient in the liquid in Eq. (4) consists of a pressure gradient due to capillary forces and pressure gradients due to viscous drag forces.

For the given pipe element of length dz

$$\frac{dP_{\text{cap}}}{dz} = \frac{dP_{\text{V}}}{dz} - 2\sigma \frac{d}{dz} \left[\frac{1}{R(z)} \right]. \tag{6}$$

The pressure gradient due to viscous forces is determined according to Darcy's Law:

$$\frac{dP_{\text{fr}}}{dz} = \frac{\mu_{\text{L}}}{K \rho_{\text{L}}} G_{\text{L}}(z). \tag{7}$$

The system of equations (3), (4), and (5) is solved, with (6) and (7) taken into account, separately for each zone of the heat pipe. Combining (3) and (5), we obtain

$$G_{\text{L}}(z) + C = \pm \frac{Q}{\varepsilon F_{\text{wick}} r} \cdot \frac{z}{L}, \tag{8}$$

where $C = 0$ for the condenser and $C = -Q/\varepsilon F_{\text{wick}} r$ for the evaporator. Inserting (8) into (4), then integrating and adding up the results of integration for each zone, we obtain an expression for the heat transmitting capability of a heat pipe based on the properties of the working liquid, on the wick structure parameters, on the thermal mode of operation, and on the geometry:

$$Q_{\text{G}} = \frac{4K\varepsilon F_{\text{wick}} r \dfrac{\sigma(t_{\text{e}})}{R(L)}}{\dfrac{\mu_{\text{L}}(t_{\text{e}})}{\rho_{\text{L}}(t_{\text{e}})} L_{\text{e}} + 2\dfrac{\mu_{\text{L}}(t_{\text{T}})}{\rho_{\text{L}}(t_{\text{T}})} L_{\text{T}} + \dfrac{\mu_{\text{L}}(t_{\text{c}})}{\rho_{\text{L}}(t_{\text{c}})} L_{\text{c}} + \dfrac{64\mu_{\text{V}} L}{\rho_{\text{V}} d_{\text{V}}^{2}} \cdot \dfrac{\varepsilon F_{\text{wick}}}{F_{\text{V}}}}, \tag{9}$$

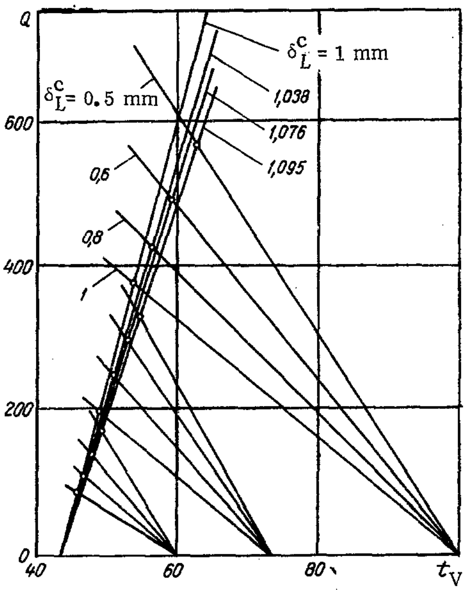

Fig. 1. Rate of heat transfer in a heat pipe vs temperature of the evaporator wall, at various thicknesses of the wet wick.

where R(L) denotes the radius of curvature of the meniscus at the evaporator end (z = 0 at the pipe end on the condenser side).

Method of Designing Heat Pipes for Operation in the Evaporation Mode. The purpose of these calculations is to determine the individual parameters of a heat pipe, namely: the vapor temperature and pressure, the temperature of condenser and evaporator walls, the pipe and wick geometry, to select the proper working liquid, to estimate the heat transmitting capacity of a pipe, etc.

The working liquid was selected here on the basis of thermal conditions in the pipe, compatibility between that liquid, the wick material, and the pipe wall, on the basis of the transmitted power, etc. The wick selection was made on the basis of earlier listed factors. The problem of optimizing the wick dimensions was discussed in [2]. The geometrical dimensions of a heat pipe are more often governed by structural considerations. By the method proposed here one can estimate the operating characteristics of a heat pipe of a given geometrical design with wick and working liquid appropriately selected.

For illustration, we will show the design of a cylindrical heat pipe L = 1000 mm long. The evaporator length is L_e = 150 mm and the condenser length is L_c = 800 mm. The wick is three layers of stainless steel (grade ChMTU-4-7-66) interwoven in a criss-cross pattern. The wick thickness is δ_{wick} = 1 mm and the longitudinal permeability is $K = 4.53 \cdot 10^{-10}$ m^2. The cell dimensions are 0.14-0.2-0.4 mm. The meshes are distributed as follows: those with small cells are located on the side of the tube wall, those with large cells are adjacent to the vapor. The body of the heat pipe is seamless, made of stainless steel tubing with diameters d_{in}/d_{out} = 23.7/24.5. The working liquid is distilled water. The heat from the condenser is carried away to maintain its wall temperature at t_{wall}^c = 45°C. The temperature of the condenser wall is generally determined from the external operating conditions at the condensation of a heat exchange and the proposed thermal flux level.

Assuming various temperatures of the evaporator wall, we determine the corresponding vapor temperatures at various penetration depths of the liquid into the evaporator wick, in accordance with the equation of heat conduction

$$Q_T^e = \frac{\lambda}{\delta_L^e}\left(t_{wall}^e - t_V\right) F_e \text{ for the evaporator,} \tag{10}$$

$$Q_T^c = \frac{\lambda}{\delta_L^c}\left(t_V - t_{wall}^c\right) F_c \text{ for the condenser,} \tag{11}$$

where δ_L^e and δ_L^c denote the thicknesses of the liquid layers inside the evaporator and inside the condenser, respectively. (When the wall and the wick cannot be considered flat, it becomes necessary to use the equations of heat conduction through a cylindrical layer.)

The following equality must be satisfied under steady-state conditions in a heat pipe:

$$Q_T^e = Q_T^c = Q_T. \tag{12}$$

In order to simplify the calculations, one plots the graphs of $Q_T = f(t_{wall}^e, \delta_L^e, t_V)$ as shown in Fig. 1. The thermal conductivity of a wet wick designed according to the earlier specifications is assumed equal to the thermal conductivity of the liquid.

The relation between δ_L^e and δ_L^c is found by simple geometrical analysis:

$$\delta_L^c = \delta_{wick} + \left(\delta_{wick} - \delta_L^e\right)\frac{L_e}{L_c}. \tag{13}$$

On the basis of Fig. 1, then, one plots the relation $Q_T = \varphi(\delta_L^e)$ for various temperatures of the evaporator wall (Fig. 2), in accordance with the equation of heat conduction. Under steady-state conditions $Q_T = Q_e$,

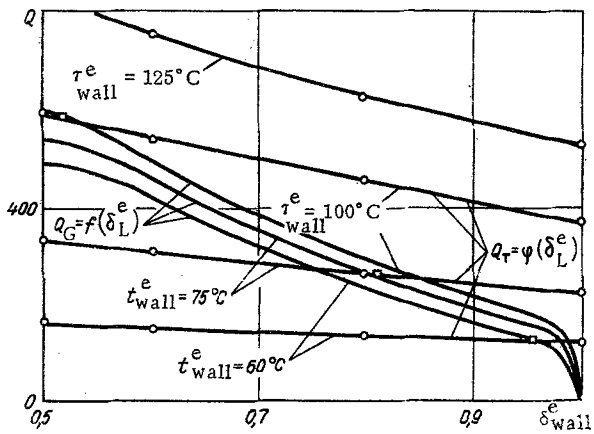

Fig. 2

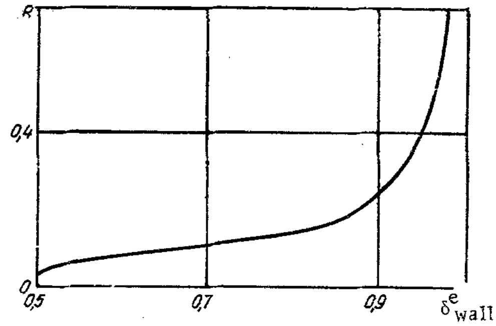

Fig. 3

Fig. 2. Determining the points of steady-state operation for a heat pipe: t^{c}_{wall} = 45°C, Q (W), δ (mm).

Fig. 3. Curvature radius of liquid meniscus vs thickness of a wet wick. Wick 0.14-0.20-0.40 mesh sizes.

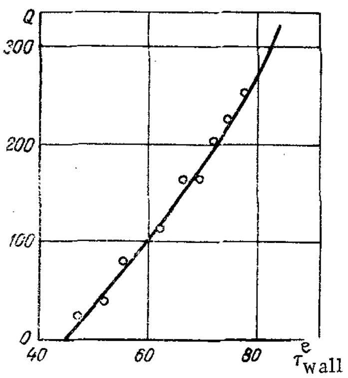

Fig. 4. Heat transmitting capability of a heat pipe L = 1000 mm long vs temperature of the evaporator wall. The line represents calculated values, the points represent test data.

i.e., the quantity of heat supplied to the evaporator by transmission must be equal to the quantity of heat supplied to it through the pipe. The value of Q_T is determined by the hydrodynamics of liquid and vapor flow.

In order to plot the $Q_G = \psi(\delta^e_L)$ curve, one makes use of formula (9) with R(L) varying as a function of the depth of liquid penetration δ^e_L in the evaporator. The relation $R = \eta(\delta^e_L)$ can be found from Eq. (2) by a simple geometrical solution of the problem. The trend of this relation is shown in Fig. 2 for the example given here. The intersection points between $Q_G = f(\delta^e_L)$ and $Q_T = \varphi(\delta^e_L)$ at respective temperatures of the evaporator wall are the points of steady-state operation. Indeed, let us assume that $Q_G > Q_T$ for a given δ^e_L, i.e., more liquid is supplied to the evaporator than evaporates from it. This will result in a deeper penetration δ^e_L and a larger radius R of the liquid meniscus. Capillary forces will begin to decrease, and the supply of liquid to the evaporator will diminish until equilibrium between supplied and evaporated liquid has been restored. And conversely, if at any instant of time $Q_T > Q_G$, more liquid will evaporate than is supplied, the liquid will penetrate deeper into the evaporator wick with a resulting decrease in the capillary radius. This will cause the capillary forces to increase and, consequently, the flow rate of liquid across the wick to increase until equilibrium has been reached. This latter condition is possible when $Q_T < Q_{Gmax}$, however, which corresponds to a minimum capillary radius or, in this given example, to the dimension of the finest mesh. Otherwise the evaporator wall would become dry. The steady-state points of operation for a heat pipe in Fig. 2 determine its performance characteristics. The latter can be represented in terms of the $Q = \gamma(t^e_{wall})$ curve for t^c_{wall} = const (in our example t^c_{wall} = 45°C), as shown in Fig. 4.

In Fig. 4 are also shown test points for the described heat pipe. Evidently, the calculated results agree well with the test data.

It is to be noted, in conclusion, that the evaporation limit can be determined from the temperature drop which corresponds to the start of boiling [5]:

$$\Delta T_c = CP^n_s, \tag{14}$$

with C = 10.25 and n = −0.33 for water, C = 17 and n = −0.28 for ethyl alcohol.

NOTATION

P is the pressure;
σ is the surface tension;

R	is the radius of curvature;
i	is the number of mesh layers;
V	is the volume;
ε	is the porosity;
F	is the cross section area;
θ	is the wetting angle;
G	is the reduced mass flow rate;
d	is the diameter;
ρ	is the density;
h	is the enthalpy;
Q	is the power;
L	is the length;
z	is the axial coordinate;
μ	is the dynamic viscosity;
K	is the permeability;
r	is the heat of evaporation;
t	is the temperature;
ν	is the kinematic viscosity;
δ_{wick}	is the thickness of the wick;
λ	is the thermal conductivity

Subscripts and Superscripts

V	denotes vapor;
L	denotes liquid;
T	denotes transport zone;
G	denotes capability;
cap	denotes capillary;
wick	
wall	
fr	denotes friction;
e	denotes evaporator;
c	denotes condenser.

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